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MATHEMATICAL MODELS OF ELECTROMAGNETIC WAVE SCATTERING BY TWO-ELEMENT STRIP GRATING WITH A PERPENDICULARLY MAGNETIZED GYROTROPIC MEDIUM

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ABSTRACT

Two mathematical models are proposed for analyzing a linearly polarized plane wave scattering by a strip grating placed on isotropic-gyrotropic media interface in the case of oblique incidence. the first mathematical model is based on reducing the original boundary value problem to the Riemann-Hilbert problem. The second model is based on reducing the same problem to a singular integral equation of the first kind with Cauchy kernel and its numerical solving by the discrete singularities method.

In this paper, the results obtained in [1-3] are generalized for the case of periodic structure consisting of two strips of different widths per period (two-element grating). This results in richer diffraction phenomena in comparison to simple grating because of additional control parameters. Moreover, unlike papers [1-3] here the case of oblique incidence of an H-polarized plane wave on a two-element grating is considered. The center of coordinate system is chosen in the middle of one of the strips.

The following set of dual series equations is mathematical model of a structure

$$\sum_{n} A_n \gamma_{n1} e^{in\varphi} = \gamma_{01}, \quad \theta_m < \varphi < \theta_m^{(1)};$$ (1)

$$\sum_{n} A_n r_n e^{in\varphi} = -p_0, \quad \theta_m^{(1)} < \varphi < \theta_{m+1}^{(1)},$$ (2)

where $\gamma_{n1} = \sqrt{k_0^2 n_1^2 - h_n^2}$, $\quad n_1 = \sqrt{\varepsilon_1 \mu_1}$, $\quad h_n = k_0 n_1 \sin\zeta + \dfrac{2\pi}{l} n$,

$$r_n = 1 + \frac{\gamma_{n1}}{\varepsilon_1 (R\gamma_{n2} - iLh_n)}, \qquad \gamma_{n2} = \sqrt{k_0^2 \varepsilon_\perp \mu_\parallel - h_n^2}, \quad \varepsilon_\perp = \frac{\varepsilon^2 - \varepsilon_a^2}{\varepsilon}$$

$$R = \varepsilon_\perp^{-1}, \quad L = -\frac{\varepsilon_a}{\varepsilon^2 - \varepsilon_a^2}, \qquad k_0 = \frac{2\pi}{\lambda},$$

$$p_0 = 1 - \frac{\gamma_{01}}{\varepsilon_1 (R\gamma_{02} - iLh_0)}, \quad \varphi = \frac{2\pi}{l} y, \quad \theta_m = 2\pi \frac{y_m}{l}, \quad (m = 1,2),$$

l is the grating period, d is the slot width, λ is the wavelength, ζ is the incidence angle.

Denote $\widetilde{A}_n = A_n r_n + p_0 \delta_{0n}$, and δ_{0n} for the Kronecker symbol.

Then initial set of dual series equations takes the form:

$$\sum_{n} \widetilde{A}_n e^{in\varphi} = 0, \qquad \theta_m < \varphi < \theta_m^{(1)};$$ (3)

$$\sum_{n>0} n\tilde{A}_n \frac{\eta_{n1}}{r_n^+} e^{in\varphi} + \sum_{n<0} |n| \tilde{A}_n \frac{\eta_{n1}}{r_n^-} e^{in\varphi} = (1 - \frac{\tilde{A}_0 - p_0}{r_0}) \kappa_1 \cos\zeta ,$$

$$\theta_m^{(1)} < \varphi < \theta_{m+1}^{(1)}, \qquad (4)$$

$$\sum_{n\neq 0} \tilde{A}_n e^{in\delta_m} = -\tilde{A}_0 , \qquad \theta_m^{(1)} < \delta_m < \theta_{m+1}^{(1)}, \qquad (5)$$

The set of equations (3)-(5) can be reduced to non-homogeneous conjugation problem (Riemann-Hilbert's problem) with a complex-valued coefficient in the case of account of dissipative losses.

To calculate matrix elements of final matrix equation. it is necessary to introduce poly-

nomials $Q_n(u_m, \rho)$ [3]. where $\rho = \dfrac{\ln|G|}{2\pi}$. $u_m = \cos\theta_m$, (m=1,2).

In the case of multi-element gratings. an efficient numerical-analytical method for solving these dual series equations was suggested in [4]. The method consists in reducing them to a singular integral equation of the first kind with the Cauchy kernel on the set of segments, and its following solution by the method of discrete singularities [4,5]. Integral equation is of the following form

$$\frac{1}{\pi}\int_L \frac{F(\xi)}{\xi - x}d\xi + \frac{1}{\pi}\int_L K(x,\xi)F(\xi)d\xi = f(x). \quad x \in L \qquad (6)$$

where $L = \bigcup_{q=1}^{m}(a_q, b_q)$, $-\infty < a_1 < b_1 < ... < a_m < b_m < +\infty$;

$f(x)$, $x \in \overline{L}$; $K(x,\xi)$, $x \in \overline{L}, \xi \in \overline{L}$ are known smooth functions, and function

$F(\xi), \xi \in L$ is sought in the functional class whose restriction on interval (a_q, b_q):

$$F_q(\xi) = F(\xi), \qquad a_q < \xi < b_q, q = 1,...,m$$

can be represented in the form

$$F_q(\xi) = \frac{v_q(\xi)}{\sqrt{(\xi - a_q)(b_q - \xi)}}, \qquad a_q < \xi < b_q,$$

where $v_q(\xi)$, $\xi \in [a_q, b_q]$ is a smooth function.

The sought function $F(\xi), \xi \in L$ satisfies additional conditions, which in general case are of the following form:

$$\frac{1}{\pi}\int_L S_p(\xi)F(\xi)d\xi = C_p, \quad p = 1,...,m, \qquad (7)$$

where $S_p(\xi)$, $\xi \in [a_p, b_p]$ is a known smooth function. and C_p is a known constant.

In conclusion we shall present the discrete mathematical model that is a set of linear algebraic equations for numerical solution of the integral equation (6) with additional condition (7).
Denote

$$t_i^n = \cos\frac{2i-1}{2n}\pi, \quad i = 1,...,n; \qquad t_{0j}^n = \cos\frac{j}{n}\pi, \quad j = 1,...,n-1;$$

$$g_k(\tau) = \frac{b_k - a_k}{2}\tau + \frac{b_k + a_k}{2}; \quad \xi_{qi}^{n_q} = g_q(t_i^{n_q}), \quad i = 1,...,n_q; q = 1,...,m$$

$$\chi_{pj}^{n_p} = g_p(t_{0j}^{n_p}), \quad j = 1,...,n_p - 1; p = 1,...,m$$

To calculate approximate values $\left\{v_{qn_q}(\xi)\right\}_{q=1}^{m}$ of the desired functions $v_q(\xi)$, $q = 1,...,m$ in principal points $\left\{t_i^{n_q}\right\}_{i=1}^{n_q}$, we have a set of linear algebraic equations (where $R(x,\xi) = \frac{1}{\xi - x} + K(x,\xi)$)

$$\sum_{q=1}^{m}\sum_{i=1}^{n_q} R(\chi_{pj}^{n_p}, \xi_{qi}^{n_q}) v_{qn_q}(\xi_{qi}^{n_q})\frac{1}{n_q} = f(\chi_{pj}^{n_p}), \quad j = 1,...,n_p - 1; \quad p = 1,...,m,$$

$$\sum_{i=1}^{n_p} S_p(\xi_{pi}^{n_p}) v_{pn_p}(\xi_{pi}^{n_p})\frac{1}{n_p} = C_p, \quad (j = n_p), \quad p = 1,...,m$$

The values of the physical characteristic of scattered field,

$$H = \int_L H(\xi)F(\xi)d\xi = \sum_{q=1}^{m}\int_{a_q}^{b_q} H_q(\xi)v_q(\xi)\frac{d\xi}{\sqrt{(\xi - a_q)(b_q - \xi)}}.$$

are expressed in terms of the functions $v_q(\xi)$, $\xi \in [a_q, b_q]$, $q = 1,...,m$, where $H_q(\xi)$, $\xi \in [a_q, b_q]$ are known functions.

Approximate values of

$$H_{\bar{n}} = \sum_{q=1}^{m}\sum_{i=1}^{n_q} H_q(\xi_{qi}^{n_q})v_{qn_q}(\xi_{qi}^{n_q})\frac{1}{n_q}, \quad \bar{n} = (n_1,...,n_m)$$

are calculated in numerical experiments.

Obtained results can be applied in the design and elaboration of various devices containing periodic structures with ferrite substrates or in plasma.

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